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Empirical and Simplified Models
of the Infiltration Process¹

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We would probably be tempted to say we are in an era of unprecedented infiltration research activity. First, though, we might recall a statement in the report of the Committee on Infiltration 1945-1946 of the American Geophysical Union. A paper by H. L. Cook (1946) included in the committee report began with the sentence, "Some future historian of the development of scientific hydrology will probably be tempted to call the present period the 'era of infiltration'." Obviously, this "era" will continue for the foreseeable future or as long as research continues in scientific hydrology.

In this brief report, a sample of reported infiltration research was selected to stimulate our discussions in the areas of

- the rainfall infiltration problem
- infiltration models
 - * empirical equations
 - * approximations of more rigorous flow equations
 - * equations derived from simplifications of the physical system
- testing and comparison of models
- estimation of model parameters
- watershed retention.

The Rainfall-Infiltration Problem

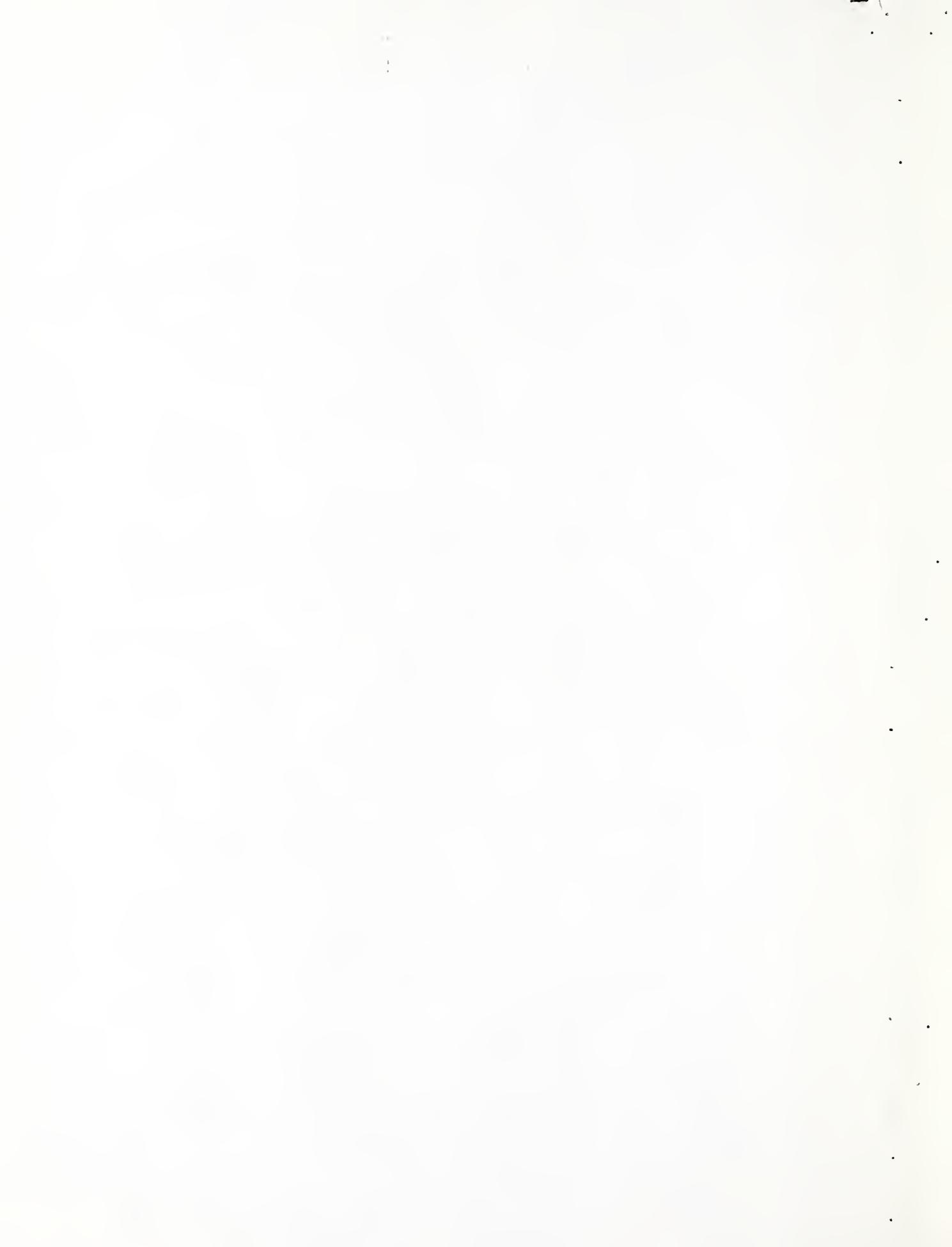
Most reported research since the Mein and Larson (1971) studies recognize the rainfall-infiltration process to be rainfall-rate controlled until surface saturation (ponding) occurs, and after ponding being controlled by the soil "hydraulic" properties. The work of Smith (1972, 1973) and Smith and Parlange (1977) also derived models which predict ponding time and infiltration after ponding. Childs (1969) described the infiltration process--"the infiltration rate may be

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regarded either as the consequence of the hydraulic conductivity and the potential gradient at the surface in accordance with Darcy's Law or alternatively as the rate of increase of the total amount of water stored in the soil profile." (italics mine) With the latter view, rainfall infiltration is simply handled by the relationship between infiltration rate and the corresponding accumulated infiltration at that rate, which then treats the pre- and post-ponding infiltration as a continuum. Thus, any infiltration rate equation and its integral can be applied to any rainfall rate-time distribution without regard, of course, to actual clock time. Questions of breaks within the rainfall distribution and subsequent redistribution are obviously important aspects still to be considered.

Infiltration Models (Algebraic and Empirical)

Most infiltration equations (those models that can be written in closed form) can be considered algebraic. Empirical is used in the sense that the equation parameters are estimated by some statistical procedure applied to time-infiltration (rate or amount) data. Some of these algebraic equations are, of course, physical based equations that can be applied empirically.

The problem with this approach is that every application is a model calibration problem utilizing infiltration or watershed data. There is always the hope that after sufficient study, parameter prediction equations will be developed. Unfortunately, this hardly ever happens.

Infiltration models that are algebraic or can be used in the algebraic and empirical sense include those developed by Brutsaert (1977), Green and Ampt (1911), Holton (1961), Overton (1964), Philip (two parameter, 1959), Smith (1972), Horton (1940), and Kostiakov (1932). This writer would recommend the Philip (two parameter) or the Green and Ampt model as an algebraic infiltration equation if this approach is taken.

Infiltration Models (Approximations)

Approximation is used here in the sense of an approximation to the more rigorous flow equations. These models would include Brutsaert (1977), Philip (1969), Smith (1972, an empirical approximation), and Smith and Parlange (1977). Parlange (1971) derived an infiltration equation valid for all times, which for finite times agreed with the Philip solution. The Philip Two-parameter model has parameters, sorptivity and effective conductivity, that can be estimated from soil-water data. The Smith and Parlange models, depending on the behavior of K near saturation, appears to have applicability over a wide range of soil and hydrologic situations and is shown to give results similar to the Green and Ampt model.

Infiltration Models (Simplification)

Simplification is used here in the sense that the physical system has to be simplified. Of course, the soil system must be simplified in varying degrees for all formulations.

The Green and Ampt equation (1911) is derived for a simplified soil physical system. A "vast" amount of literature is available reporting on its use, comparisons with other models, and evaluating its utility to model the infiltration process. Philip (1957) derived the Green and Ampt for a soil that the diffusivity function may be represented as a delta function.

An infiltration equation was derived for a somewhat approximate system by Morel-Seytoux and Khanji (1974), however, it was used mainly to formulate an interpretation for the Green and Ampt effective capillary pressure parameters. Parlange (1975) assumed that D and $dK/d\theta$ are delta functions and derived a more exact type Green and Ampt model.

Testing and Comparisons of Models

Bouwer (1969) utilized the Green and Ampt equation for ponded infiltration into a layered soil (decreasing conductivity). Appeared to give reasonable results.

Fok (1976) utilizing a series expansion of the Green and Ampt equation derived the Philip two-parameter equation. Time limits are given for insuring a difference less than 17% or less than 5%. Parameter relationships are given for equation equivalence.

Li, Stevens and Simons (1976) derived both an explicit and an implicit form of the Green and Ampt equation for a homogenous soil.

Mein and Larson (1971) utilized actual soil data (5 soils) to compare Green and Ampt with the Richards equation. Depending on soil type, the comparison was fair to good. The rainfall-infiltration problem was solved in two stages, time to ponding and after ponding. A constant rainfall rate was assumed and the soil was assumed to be homogenous.

Onstad et al. (1973) compared Green and Ampt with infiltrometer (ponded) data. Comparisons were poor to good. However, parameter estimations of the Green and Ampt equations are questionable.

Parlange (1975) compared his improved Green and Ampt equation with the original. Maximum deviation was 20%.

Philip (1957) compared Horton, Kostiakov, Green and Ampt, and Philip equations (two parameter). The Horton and Kostiakov equations, in general, failed; Horton was worst. Green and Ampt and Philip equations were equally good.

Skaggs et al. (1969) compared Green and Ampt, Horton, Philip (two parameter) and Holton equations by fitting to rainulator-derived infiltration data. Horton and Holton (three-parameter equations) gave highest correlations. Green and Ampt and Philip gave essentially equal correlation. This writer would question the form of the Green and Ampt equation used for fitting. Also open to question is whether a 35-foot plot should be used as an infiltrometer even after using overland flow routing procedures to correct for runoff travel time.

Swartzendruber and Youngs (1974) numerically compared the Green and Ampt equation and the two-term Philip equation. The two were within 15.1 percent. The Philip two-term equation was recommended.

Whisler and Bouwer (1970) compared the Green and Ampt equation with the Philip two-parameter equation with numerical data. Concluded that Green and Ampt was the easiest to use and gave reasonably accurate results.

Dooge (1973) compared analytically physical-based and empirical infiltration equations under various simplifying assumptions. For small time he concluded that the various models correspond to the Kostiakov equation with an exponent of 1/2. He also discusses the usefulness in simulation studies of the relationship between the infiltration rate and the volume of actual or potential infiltration.

Smith (1976) theoretically discussed the Green and Ampt, Philip's two-parameter, SCS equation, Holton equation, and Mein and Larson time to ponding equation. He recommends Green and Ampt or Philip's equation for an empirical engineering application. He considers physical estimation of parameters impractical. He considers Holton and SCS models unsatisfactory.

The recent work by Smith and Parlange (1977) opens new insights into the Green and Ampt equation and defines more precisely the physical situation for which it applies. They derived alternative models depending on the behavior of K near saturation. Slow variations of K near saturation led to the Green and Ampt model. Exponential variation of K near saturation led to the Parlange (1971) model. Examples appeared to substantiate the applicability of the Green and Ampt model to hydrologic investigations.

Rawls (1976), utilizing a Purdue infiltrometer, measured infiltration rates for 11 Coastal Plains soils near Tifton, Georgia. An empirical comparison of the Horton, Holton, Phillip, Green and Ampt, and Snyder infiltration equations was made. Horton's and Synder's equations being four and three parameter equations gave the highest correlation (0.86 and 0.95, respectively). The Phillip and Green and Ampt (two parameters) equations were similar with a correlation of 0.82. The Holton equation had the lowest correlation, 0.74.

Model Parameter Estimation

If infiltrometer data, i.e. infiltration rates and/or amounts, as a function of time are available, then a statistical estimate of any of the model parameters could probably be attempted, Skaggs et al. (1969). Some

limited experiences by the writer (Brakensiek and Onstad, 1977) would indicate that site to site parameter variation is very great. Spatial distributional studies are needed.

Very similar to the above is the estimation of infiltration parameters which are components of total watershed models. Sensitivity analyses are very valuable for identifying the overall influence of infiltration parameter errors on, say, runoff estimation.

The estimation of parameters in infiltration models which have a base in the soil physical system should proceed from measurable soil properties. For the Green and Ampt and Philip's models, parameter estimation procedures that have been reported are listed:

Green and Ampt

Bouwer (1969):

- Effective capillary pressure, H_f , is the water entry value of the soil and can be estimated as 0.5 of the air-entry value.
- Effective hydraulic conductivity, K_o , can be described as the "rewet" hydraulic conductivity and can be taken as 0.5 of the saturated conductivity.
- The fillable porosity, n , can be obtained from volumetric water content before and after wetting.

Swartzendruber and Youngs (1974):

- K_o is the near saturated hydraulic conductivity.
- H_f is related to sorptivity; i.e., $H_f = S^2/2K_o n$.
- Fillable porosity is the difference between nominal saturation and the initial soil-water content.

Onstad et al. (1973):

- H_f taken as the pressure head where the time *vs.* measured pressure head function flattened out.
- K_o taken as $1/2 K_s$
- n calculated as $n = 0.86(1 - d/2.65) - W_a$; where d = bulk density and W_a = antecedent soil water

Yu-Si Fok (1975):

- Green and Ampt identical with Philip two-parameter when,

$$H_f = S^2/2 n K$$

$$K_o = \text{hydraulic conductivity in the transmission zone}$$

$$n = \text{product of total porosity and net increment of the degree of saturation}$$

Smith and Parlange (1977) - $\psi_{avg} = \frac{\int_{\psi_i}^{\psi_o} K_r d\psi}{\int_{\psi_i}^{\psi_o} 1} \quad , \text{ where } \psi_{avg} \text{ is the average capillary tension across the wetting front}$

Mein and Larson (1971): - $S_{avg} = \frac{\int_0^1 S d k_r}{\int_0^1 1}$

K_s - saturated hydraulic conductivity

Brakensiek (1977) evaluated several schemes for estimating the capillary suction parameter utilizing the Mein and Larson data (1971). Estimated values were compared with those calculated by H. J. Morel-Seytoux (1974). Reasonable agreement was shown by utilizing the Brooks and Corey (1964) relative conductivity function for sorption and also by calculation of relative conductivity from soil water characteristics (Jackson, 1972).

Morel-Seytoux (1974) derived an equation for H_f . Comparisons indicated that the Mein and Larson estimation procedure is better than the Whisler and Bouwer procedure.

Philip's Two Parameter

Youngs (1968): - (In analogy with Green and Ampt)

$$(\text{Sorptivity}) \quad A = \sqrt{2n K_s \text{Pa}}$$

$$B = 2/3 K_s$$

Whisler and Bouwer (1970): - Parameters were determined by statistical fitting of equation to experimental data.

Smith and Parlange (1977): - Rough estimate of sorptivity:

$$S = \sqrt{2\psi_{avg} n K}$$

Brutsaert (1976): - Formulations for the sorption parameter

(1976):

Watershed Retention

Snyder (1971) introduced a so-called watershed retention function. He considers this approach as macro-scale as compared to the point infiltration approach; i.e., a micro-scale approach to rainfall excess. His contention is that the macro approach is feasible, whereas the spatial integration of point infiltration is not yet feasible. The parameters of the retention model were estimated by an optimization process applied to plot data.

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